Today, I will take the question I gave you in the last post (and that is all we will tackle today!) It is a question from GMAT prep test so it is quite indicative of the tricky questions you might get on actual GMAT. Solving the question takes less than two minutes since the calculations required are negligible. However, if you start calculating the actual value, you could end up spending many painful minutes before giving up. I implore you to always remember that GMAT does not give you calculation intensive questions. Since they do not provide you with an HP12C, there will always be a logical solution -– you will just need to think a little harder! Let's get going then…

Question 3: For every integer m from 1 to 10 inclusive, the mth term of a certain sequence is given by [(-1)^(m+1)]*[(1/2)^m]. If T is the sum of the first 10 terms in the sequence, then T is

(A) greater than 2
(B) between 1 & 2
(C) between 0.5 and 1
(D) between 0.25 and 0.5
(E) less than 0.25

Solution:

I agree that the expression for the mth term looks daunting. But as we discussed last week, the first step in a sequence question should be to write down the first few terms of the sequence. So let's do that and see what we get.

We get the first term by putting m = 1

First term = [(-1)^(1+1)]*[(1/2)^1] = **1/2** (Not so bad, eh?!)

Second term = [(-1)^(2+1)]*[(1/2)^2] = -1/2^2 = **-1/4**

Third term = [(-1)^(3+1)]*[(1/2)^3] = 1/2^3 = **1/8**

Do we see a pattern?

The tenth term will be [(-1)^(10+1)]*[(1/2)^10] = -1/2^10 = **-1/1024** (You don't need to calculate this of course. You can keep it as 2^10. I do suggest though that you should be good with the first 10 powers of 2, first 6 powers of 3, first 4 powers of 4 and first 3 powers of 5 to 10.)

The sequence looks like this: 1/2, – 1/4, 1/8, – 1/16, …

T = 1/2 – 1/4 + 1/8 – 1/16 +…+ 1/512 – 1/1024

Of course GMAT doesn't expect us to calculate this. One could end up wasting a lot of precious time if one did. The trick is to know that we need to figure out the answer using some shrewdness. Fortunately (or unfortunately), GMAT software rewards cunning and craft!

The problem is that we have positive and negative terms so it is very hard to say what the value of T will be. But the terms are not random. There is a positive term followed by a negative term which is then followed by a positive term and so on. Also, every subsequent term is smaller than the previous term (in fact it is a Geometric Progression but we don't need to know that to solve this question. Nevertheless, we will take up GP too in a couple of weeks).

We need to create some uniformity so that we can deduce something about T. We have 10 terms. If we couple them up, two terms each, we get 5 groups:
T = (1/2 – 1/4) + (1/8 – 1/16) + … + (1/512 – 1/1024)

Tell me, can we say that each group is positive? From a larger number, you are subtracting a smaller number in each bracket. The first number is greater than the second number in each group e.g. 1/2 is greater than 1/4, therefore, (1/2 – 1/4) = 1/4 i.e. a positive number
Similarly, (1/8 – 1/16) = 1/16, again a positive number.

This means T = 1/4 + 1/16 +…. (all positives)
Definitely this sum, T, is greater than 1/4 i.e. 0.25

So we can rule out option (E). But we still have to choose one out of the four remaining options.

Now, let's group the terms in another way.

T = 1/2 + (- 1/4 + 1/8) + (- 1/16 + 1/32) … – 1/1024

You leave out the first term and start grouping two terms at a time. The last term will be left alone too! You will be able to make four groups with the 8 terms in the middle.

Now look closely at each group: The first term is a negative number with a higher absolute value while the second term is a smaller positive number so the sum will give you a negative number, e.g.:

(-1/4 + 1/8) = -1/8

(-1/16 + 1/32) = -1/32 etc

This means T = 1/2 – 1/8 – 1/32 … -1/1024

All 4 of the groups will give you a negative number and the last term is also negative. Since the first term is 1/2 i.e. 0.5, we can say that the sum T will be less than 0.5 since all the other terms are negative.

So the sum, T, must be more than 0.25 but less than 0.5.

Answer has to be option (D).

There are other ways of arriving at the answer here. We will look at it from the Geometric Progression perspective some time later.

I will leave you now with a question I saw somewhere once. Let's see if you can use your craft to arrive at the answer in a minute! (absolutely do-able)

Question:

In the infinite sequence A, the nth term, A(n), is given by x^(n-1) + x^n + x^(n+1) + x^(n+2) + x^(n+3) where x is a positive integer constant. For what value of n is the ratio of A(n) to x(1+x(1+x(1+x(1+x)))) equal to x^5?

(A) 8
(B) 7
(C) 6
(D) 5
(E) 4